Default Risk, Coordination Failure, and Emerging Economy Business Cycles

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Abstract

Business cycles in emerging market economies are systematically associated with countercyclical changes in their country spreads: namely, in bad times, countries borrow less at higher spread and default more frequently. To examine this prominent aspect of emerging countries, I develop a small open economy model of sovereign default in a dynamic stochastic general equilibrium setting. In this model, defaults arise not only from bad output shocks, but also from coordination failures among international lenders. When a country’s level of debt lies within a certain output-contingent interval, a loss of lenders’ confidence in the country’s repayment ability leads to a debt roll-over failure, triggering a self-fulfilling default. The default probability explicitly depends on the strategic incentives and the beliefs of market participants. In a quantitative analysis of Argentina, the model accounts for several features of the economy: defaults are only weakly correlated with outputs, net exports are highly volatile, and consumption fluctuates significantly more than output. The model also predicts that the country’s level of borrowing is sensitive to small changes in the probability of coordination failure.

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1 Introduction

It is well documented that business cycles in emerging economies are typically far more volatile than in developed economies\(^1\): net exports are highly volatile and strongly countercyclical and consumption fluctuates substantially more than output. Furthermore, emerging market countries regularly default on their foreign currency debts, often in coincidence with sudden reversals of market sentiment. Recent empirical studies show that this high cyclical volatility and the frequent external debt crises in emerging countries are systematically associated with countercyclical changes in the country spreads\(^2\): in bad times, emerging markets borrow less at higher spread and default more frequently, and vice versa. These interrelated aspects of emerging economies call for a model that allows a joint analysis of business cycles and country spread dynamics.

To examine this prominent link between sovereign defaults, country spreads, and macroeconomic fluctuations in emerging countries, I develop a small open economy model of sovereign default in a dynamic stochastic general equilibrium setting. In this model, defaults can occur not only from bad output shocks, but also from coordination failures among international investors. The probability of default explicitly depends on the strategic incentives and the beliefs of market participants. In the theoretical part of this paper, I study how economic fundamentals within the model economy determine these incentives. In a quantitative exercise on Argentina, I analyze the extent to which this default probability can account for the above characteristics of emerging economy business cycles.

This paper builds on the reputation model of sovereign default by Eaton and Gersovitz (1981) and on the recent applications of their approach by Aguiar and Gopinath (2006), Arellano (2005), and Yue (2005). The model features a benevolent government maximizing a representative domestic household’s utility, and a large group of risk-neutral international investors. In each period, the government receives a stochastic endowment and sells one-period non-contingent discount bonds to international investors. The government has the option to default on this bond, in which case it temporarily loses access to the international capital market and suffers some direct output loss. The incompleteness of the asset market structure in this model enables default to arise as an equilibrium outcome of the economy. This is because the default option in this setting effectively provides the government with partial insurance against a bad income shock, allowing higher consumption today at the cost of a higher default premium in the country spread. In equilibrium, the discount bond

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\(^1\)Aguiar and Gopinath (2004) and Neumeyer and Perri (2005)

\(^2\)Uribe and Yue (2003), and Neumeyer and Perri (2005)
price accurately reflects the government’s expected default probability and is determined to ensure that the investors are indifferent between a risky government bond and a riskless bond\(^3\).

The distinguishing aspect of this paper is that it investigates the business cycle implications of default risk in an environment in which defaults can occur not only from bad output shocks, but also from coordination failures among international investors. To study the investors’ strategic lending decision in the government bond market, I model this market as a version of a competitive auction in which each investor submits a bond purchase bid consisting of a quantity and a price. The coordination problem arises in this market because of the government’s need to roll over its debt. When the government’s level of debt lies within a certain output-contingent interval (the ‘crisis zone’), the government’s incentive to repay its old debt at the end of the period becomes dependent on the government’s sales of new bonds at the beginning of the period\(^4\). Because of the inability to commit \textit{ex ante} to a debt repayment policy, the investors’ expectations regarding the government’s repayment ability become self-fulfilling within the crisis zone. If for some reason investors believe that the government will repay its maturing debt, they purchase the newly issued bonds and the government indeed repays, validating their prior belief. If, however, investors expect the government to default, they do not make any new loans to the government and the government defaults in equilibrium, once again making the expectation self-fulfilling. Of these two equilibria, the second outcome is Pareto-dominated by the first outcome and hence constitutes a coordination failure. The investors’ expectations regarding the government’s debt repayment alternates between an ‘optimistic’ state and a ‘pessimistic’ state in a random fashion, coordinated by a perfectly observable public signal.

In this paper, the economic condition that gives rise to the coordination problem is not an artifact of a hypothetical parameter value, but rather an actual feature of an economy that we can directly calculate from the country’s GDP and external debt data. In the quantitative analysis, I calibrate the model using the data for Argentina. In particular, I choose the belief of international investors such that it is consistent with the Argentina’s GDP series and historical default frequency. The simulation results reproduce several characteristic features of the economy. First, because of the self-fulfilling crisis feature, the

\(^3\)In this paper, the 3-months U.S. Treasury Bill is considered riskless.

\(^4\)Two assumptions play crucial roles in this result. One is that each investor is “small enough” such that they cannot individually affect the equilibrium price. Another is the belief that any deviator who lends to the defaulting government can be effectively punished in some way so that no investor is willing to buy the defaulting government’s new debt at a positive price.
model exhibits a much weaker negative relationship between output and default than that predicted by previous studies. This result is consistent with recent empirical findings on the historical relationship between output and default\(^5\). Second, in line with the data, model consumption is significantly more volatile than output and net exports are consequently highly volatile. Interest rates and net exports are both countercyclical, implying that the country borrows less, and at higher cost, in bad times\(^6\). Finally, the model predicts that even a small increase in the probability of coordination failure within the crisis zone can substantially reduce government borrowing. For example, the possibility of a self-fulfilling crisis occurring at the frequency of once every 250 quarters reduces the country’s quarterly debt payment to GDP ratio by nearly 2% compared with the results of a baseline model with no coordination failure. At a quarterly interest rate of 2%, this implies a reduction in the country’s average total debt to GDP ratio of 100%. The results are robust to the risk aversion parameter, but are sensitive to the country’s cost of default.

Following a brief review on the related literature, I organize the rest of the paper as follows: In section 2, I describe the model environment and define a recursive competitive equilibrium of the economy. Then, I show how self-fulfilling crisis can occur from the investors’ coordination failure in the bond market. In section 3, I solve the model numerically and discuss its quantitative implications for the Argentina’s business cycle. Finally, section 4 concludes the paper.

### 1.1 Related Literature

The theoretical approach in this paper is closely related to multiple equilibria models of bank runs (Diamond and Dybvig (1983)) and speculative currency attacks (Obstfeld (1986, 1996), Krugman (1996), Sachs, Tornell, and Velasco (1996)) in international setting. In the context of debt repudiation, Calvo (1988) was the first one to show in a simple two-period environment with domestic nominal debt that the government’s inability to commit in advance to its monetary policy, or more specifically to its inflation target, leads to two possible Pareto-ranked equilibria when the economy is within a certain range of parameters. In one equilibrium, the government makes full repayment with zero inflation and in another equilibrium it partially repudiates its debt through inflation. The model is fully deterministic and the cost of repudiation is proportional to the size of repudiation. Extending the work\(^5\) Tomz and Wright (2006)\(^6\) This is a common attribute of models that feature stochastic endowment trend and endogenous default risk as in Aguiar and Gopinath (2006).
of Calvo (1988), Alesina, Prati, and Tabellini (1990) study a multiple equilibrium model of an infinite horizon economy in which the cost of default is a permanent drop in output. In their model, the government in equilibrium either defaults only once in the initial period or repays its initial debt and never defaults in the subsequent periods. Other papers on self-fulfilling sovereign debt crises also include Detrigiache (1996) and Lambertini (2003).

In modelling self-fulfilling debt crises, this paper draws heavily from Cole and Kehoe’s (1996, 2000) work that characterizes the multiple equilibria crisis zone and the government’s optimal policy in a dynamic general equilibrium setting. Their model features a deterministic production technology and an optimizing private sector making its own consumption and savings decision. As in this model, self-fulfilling crisis arises when the government finds itself unable to roll over its maturing debt by selling new bond at a positive price. However, because liquidity crisis driven by self-fulfilling expectation is the only source of default in their environment, the expected default probability is fully determined by the beliefs of international lenders. One contribution of this paper with respect to Cole and Kehoe (2000) is adapting their setting into a stochastic endowment economy to provide a quantitative framework for studying the business cycle implications of endogenous country spread determined by the strategic incentives and the beliefs of agents. This comes at the expense of supressing the private households as passive agents who have no access to storage technology and have to consume whatever resource the government makes available to them through its optimal foreign debt policy. In this environment, the expected default probability depends not only on the investors’ belief, but also on the current period’s endowment.

This paper is also closely related to a fast-growing literature on endogenous default risk of sovereign debt. The papers in this literature (Aguiar and Gopinath (2006), Arellano (2005, 2006), Bai and Zhang (2005), Gumus (2006), Lizarazo (2005), Yue (2005)) build on the theoretical framework of Eaton and Gersovitz (1981) on sovereign default and adopt the quantitative formulation of Chatterjee, Corbae, Nakajima, Rios-Rull’s (2005) work on consumer bankruptcy. In an infinite horizon stochastic general equilibrium setting, the models in these papers typically feature a benevolent government that maximizes the lifetime utility of domestic representative household by selling one-period non-contingent zero coupon discount bond and a continuum of financial intermediaries. In these models, the government optimally decides to default whenever the realization of endowment makes the default incentive greater than the repayment incentive. The default probability, in turn, depends on these incentives. Arellano (2005) studies a two-sector (tradable and non-tradable) version of this economy and Gumus (2006) extends the analysis to include a non-tradable denominated bond. Bai and Zhang (2005) endogenizes the value of default
option by introducing productive capital into the model and use the model to explain the lack of international consumption risk sharing. Lizarazo (2005) studies a similar model with exogenous stochastic endowment process, but with risk-averse investors. Finally, Yue (2005) develops a model that features both endogenous default probability and endogenous recovery rate on defaulted debt.

The current model builds on the work of Aguiar and Gopinath (2006) on endogenous sovereign default risk and emerging economy business cycles. Their paper develops a stochastic general equilibrium model of small open endowment economy similar to the model in this paper. In their model, however, default probability depends solely on the government’s incentive for repayment. This paper directly augments their model by incorporating the investors’ coordination problem in the bond market. As consequence, default probability in this model depends not only on the government’s repayment incentive, but also on the belief of international investors. In fact, the current model incorporates the original setting of Aguiar and Gopinath (2006) as a special case in which the beliefs of investors are always ‘optimistic’ within the crisis zone.

Finally, there exists a large body of studies started from the pioneering work of Kehoe and Levine (1993) and Alvarez and Jermann (2000) on lack of commitment problem in environments where agents have access to a full set of state-contingent claims. In these models, the availability of default option places an endogenous borrowing limit for each state of the world and the agents choose their optimal asset portfolio subject to this constraint. As result, default never arises as an equilibrium outcome of the economy as in this paper.

2 The Model

2.1 Setup

Consider a small open economy with a single non-storable good and a constant riskless world interest rate $r$. There are two types of infinitely-lived agents in this economy: the government and a measure $N$ of international investors. At the beginning of each period, the government receives a stochastic draw of the single good $y_t$ from a compact set $\mathcal{Y} \subset \mathbb{R}^+$ according to a Markov probability measure $\pi_y(y_t | y_{t-1})$. The endowment realization is perfectly observable to both types of agents. Investors on the other hand each receive a constant

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7In international setting, this environment is studied in several papers including Kehoe and Perri (2002) and Miller, Tomz, and Wright (2006).

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stream of this good every period, which I normalize to one. Along with the endowment, the government and international investors also observe an \textit{i.i.d.} random sunspot variable, $\xi_t$. The sunspot variable takes a value from the set $\Xi = \{0, 1\}$ and has a simple probability measure, $\pi_\xi(1) = \eta$ and $\pi_\xi(0) = 1 - \eta$, where $0 < \eta < 1$.

The government is a benevolent type and thus maximizes a representative domestic household’s lifetime utility

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau} u(c_{\tau}),$$

where $0 < \beta < 1$ is a constant time discount factor. The within-period utility function $u(c)$ is monotonically increasing, strictly concave, continuously differentiable, and satisfies the Inada condition, $\lim_{c \to 0} u'(c) = \infty$.

In each period, the government issues one-period zero coupon bond $B_{t+1}$ and decides whether or not to default on its maturing bond $B_t$. I denote the government’s default decision as $d_t$, where $d_t = 0$ indicates repayment and $d_t = 1$ default. A bond with negative face value $B_{t+1} < 0$ denotes a non-contingent debt contract in which, conditional on not defaulting, the government promises to deliver $-B_{t+1}$ units of consumption tomorrow and investors purchase this bond at a discount price $q_t$ today. The bond price $q_t$ depends on the government’s incentive for repayment and is determined as a function of the constant riskless rate $r$, the current endowment $y_t$, the amount of new borrowing $B_{t+1}$, and the sunspot probability $\eta$. Finally, I set the asset space $\mathcal{B} = [B, \bar{B}]$ such that $\bar{B}$ never binds in equilibrium.

International investors, indexed by $i \in \mathcal{I} = [0, N]$, are all identical and risk neutral. The individual investor has lifetime utility

$$E_t \sum_{\tau=t}^{\infty} \delta^{\tau} x_{\tau},$$

where $0 < \delta < 1$ is the investor’s discount factor and $1/\delta = 1 + r$. Hence, investors are just indifferent between consuming one unit of consumption good today and purchasing one unit of risk-free bond for consumption tomorrow.

In every period, the individual investor chooses how much new debt to buy out of his endowment. When purchasing new debt $b_{t+1}$, each investor behaves perfectly competitively and faces the budget constraint

$$q_t b_{t+1} \leq 1.$$  (1)
After purchasing $b_{t+1}$, the investor consumes his remaining endowment at the end of the period along with the proceeds from the last period’s bond purchase. Since the government has the option to default on its old debt, the investor’s available resource for consumption later in the period will depend whether or not the government repays its debt: that is, $1 - b_t + q_t b_{t+1}$ if the government repays and $1 + q_t b_{t+1}$ if it defaults.

**Timing** Let $s_t = (y_t, B_t, \xi_t)$ denote the aggregate state of the economy. The sequence of actions within each period is as follows:

1. The endowment and sunspot variables are realized: $s_t = (y_t, B_t, \xi_t)$.
2. The government, given the bond price schedule $q_t = q(y_t, B_{t+1}, \eta)$, issues $B_{t+1}$.
3. Investors, given $q_t$, purchase $b_{t+1}$.
4. The government decides whether or not to default: $d_t$.
5. Investors consume $x_t$.

A key feature of this timing\(^8\) is that the government must sell new bonds before repaying its old debt. As we shall see, this demand for liquidity is what makes coordination failure possible in this model. When the level of debt lies within a certain range of values, the government’s default decision at the end of the period becomes contingent on the investors’ purchase of new bonds at the beginning of the period. If for any reason the investors inside this range expect the government to default, then they do not purchase the newly issued bonds at any positive prices and the government optimally chooses to default, validating the investors’ expectation.

**Default Cost** As in the other similar models in the literature, default in this model incurs a temporary exclusion from international borrowing and lending and some loss of output during financial exclusion.

Upon default, the government loses its borrowing and lending privileges for a stochastic number of periods until the government eventually regains access to borrowing and lending with an exogenous return probability $0 < \psi < 1$. Bondholders can credibly punish any investor who lends to the defaulting government by seizing the government’s next period repayment to this investor\(^9\). Given this threat, no investor is willing to buy the new government debt whenever he expects the government to default on its old debt later in that

\(^8\)This timing follows Cole and Kehoe (2000).
\(^9\)This type of punishment can be implemented in a number of different ways. For example, see Neumeyer and Perri (2005) and Wright’s (2002) cheating-the-cheater strategy.
period. As I will show later, this assumption plays a crucial role in making self-fulfilling crisis possible in this model.

In addition to the temporary exclusion, the defaulting government also suffers some endowment loss of fraction $\lambda$ during the periods of financial autarky. One rationale behind this type of cost has that default often has reputation spillover effect that affects the country’s other economic activities such as its international trade (Cole and Kehoe (1998))\textsuperscript{10}.

2.2 Recursive Equilibrium

I now define a recursive equilibrium in which the government cannot commit to honor its debt contract with the investors. Within each period, the aggregate state $s = (y, B, \xi)$, the government’s policy $(B', d)(s)$, a representative investor’s policy $b'(s)$, and the bond price schedule $q(y, B', \eta)$ determine a recursive equilibrium.

**The Government** In each period, the government’s objective is to maximize the representative domestic household’s lifetime utility $v^u(s)$ by choosing the debt policy $B'(s)$ and the default decision $d(s)$ optimally.

First, suppose the government repays its debt. Taking the aggregate state $s = (y, B, \xi)$ and the bond price schedule $q(y, B', \eta)$ as given, the government chooses its debt policy $B'(s)$ such that it solves the functional equation

$$v^r(y, B, \xi) = \max_{B'} u(y + B - q(y, B', \eta)B') + \beta \int_{y \times \Xi} v^u(y', B', \xi') d\pi_{y'|y} d\pi_{\xi'},$$

(2)

where $v^r(y, B, \xi)$ denotes the government’s lifetime expected value of repaying $B$ and selling $B'$ at the discount price $q(y, B', \eta)$.

When choosing its debt policy $B'(s)$, the government knows exactly how the equilibrium bond price $q(y, B', \eta)$ depends on the current endowment $y$, its level of new debt $B'$, and the sunspot probability $\eta$. The government can also predictly perfectly what the investors’ purchasing decision $b'$ would be at this price. Notice that there is no need to impose an exogenous constraint on $B'$ to prevent Ponzi schemes since the bond price $q(y, B', \eta)$ effectively sets an endogenous borrowing limit for the government.

Suppose, on the other hand, that the government defaults on its old debt. In this case, the government immediately loses its access to international borrowing and lending and

\textsuperscript{10}In an empirical study, Rose (2002) finds that sovereign default leads to an annual 8% decline in the bilateral trade with the country’s major trading partners.
simply consumes its current endowment. In the following period, the government starts in zero net asset position. The expected value of default for the government is defined by

\[
v^d(y, B, \xi) = u(y) + \beta \int_{Y \times \Xi} v^a(y') d\pi_{y'|y},
\]

where \(v^a(y)\) denotes the value of temporary financial autarky for the government.

In the periods following default, the government either stays in autarky with probability \(1 - \psi\) or returns to international capital market with probability \(\psi\). The government also loses \(\lambda y\) each period as direct endowment cost. Taken together, these default costs imply that the value of the temporary autarky \(v^a(y)\) is a solution to the following functional equation:

\[
v^a(y) = u((1 - \lambda)y) + \beta \int_{Y \times \Xi} \left[\psi v^a(y') + (1 - \psi) v^u(y', 0, \xi')\right] d\pi_{y'|y} d\pi_{\xi'}. \tag{4}
\]

This type of default penalty has been commonly adopted in many other closely related papers such as Aguiar and Gopinath (2006) and Arellano (2005) as a simple way to model the potential cost of default that the country may experience. The assumption that the defaulting government is not only restricted from borrowing, but also from lending, is due to Bulow and Rogoff’s (1989) result. In their paper, they show that no positive amount of lending can be sustained in an environment in which the government can save at the market interest rate after defaulting. The direct output cost, on the other hand, ensures that a realistic level of international lending can be sustained in this economy.

Finally, taking the value of repayment \(v^r(s)\) and default \(v^d(s)\) as given, the government makes its static default decision by choosing the higher of these two values:

\[
v^u(s) = \max \left\{v^r(s), v^d(s)\right\}, \tag{5}
\]

where \(d(s) = 0\) if \(v^u(s) = v^r(s)\) and \(d(s) = 1\) if \(v^u(s) = v^d(s)\).

**Investors** In each period, an individual investor takes the aggregate state \(s\), his individual bond holdings \(b\), the newly issued bond \(B'\), and the equilibrium bond price \(q(s, B')\) as given and chooses his lending policy \(b'\) to maximize his lifetime expected utility \(w\).

When choosing his policy, the investor knows exactly how the aggregate lending affects the government’s choice of equilibrium policy \((B', d)(s)\) and what the equilibrium price
q(s, B') will be for each state s and B'. Hence, the investor solves the following problem:

$$w(s, b, B') = \max_{b'} x + \delta \int_{Y \times \Xi} w(s', b', B'(s')) d\pi_{y'|y} d\pi_{\xi'},$$ \hspace{1cm} (6)

subject to

$$q(y, B', \eta)b' \leq 1,$$ \hspace{1cm} (7)

$$x = 1 - (1 - d)b + q(y, B', \eta)b',$$ \hspace{1cm} (8)

$$s' = (y', B'(s), \xi').$$ \hspace{1cm} (9)

Since investors are perfectly competitive and risk-neutral, they are relatively passive agents in this model. They are willing to buy newly issued bonds as long as the bond price q(y, B', \eta) fully compensates them for the expected default risk associated with the new bonds. This implies that the equilibrium bond price schedule q(y, B', \eta) must satisfy

$$q(y, B', \eta) = \frac{1 - \phi(y, B', \eta)}{1 + r},$$ \hspace{1cm} (10)

where \( \phi(y, B', \eta) \) denotes the expected default probability consistent with the government’s equilibrium policies and is defined by

$$\phi(y, B', \eta) = \int_{Y \times \Xi} d(y', B', \xi') d\pi_{y'|y} d\pi_{\xi'}.$$

To further characterize \( \phi(y, B', \eta) \), I describe in the next section how the government’s equilibrium default decision depends on the investors’ expectation in the bond market. In equilibrium, the expected net return on the new bond equals the riskless rate r because investors lend to the government perfectly competitively.

**Recursive equilibrium** Having described the government’s and the investors’ problems, I now define a recursive equilibrium of this economy as follows:

A recursive equilibrium in this economy consists of (a) a set of value functions \( \{v^u, v^r, v^d, v^a\} \) for the government and \( w \) for the representative investor, (b) the government’s debt policy \( B' \) and default decision \( d \), (c) the representative individual investor’s lending decision \( b' \), and (d) the bond price schedule \( q \) such that

1. Given \( \{v^u, v^r, v^d, v^a\} \) and the bond price schedule \( q \), the government’s debt policy \( B' \) and default decision \( d \) solve the government’s problem, (2)-(5).

2. Given \( w \) and \( q \), the representative investor’s purchase decision \( b' \) solves the investor’s problem (6) subject to (7)-(9).
3. The bond market clears: \( B'(y, B, \xi) = b'(y, B, \xi, B, B') \).

This definition is similar to Cole and Kehoe’s (2002) notion of time-consistent equilibrium in that the government is unable to commit \( \textit{ex ante} \) to repay its debt even when repayment is strictly Pareto-superior to default. The government instead chooses a set of state-contingent policies that it will follow sequentially within each period\(^{11}\).

This equilibrium also closely resembles the equilibrium concepts in Aguiar and Gopinath (2006), Arellano (2005), and Yue (2005) on sovereign debt crisis and Chatterjee, Corbae, Nakajima, and Rios-Rull (2005) on personal bankruptcy\(^{12}\). These models all commonly feature equilibrium default probability that depend on the government’s repayment incentive. This paper extends their equilibrium concept by explicitly taking into account the coordination problem that arises in the government bond market. As result, the default probability in this model depends not only on the government’s strategic incentive, but also on the beliefs of international investors regarding the government’ repayment policy.

The number of equilibrium in this model economy could be more than one and this is true even when the government does not need to roll over its debt as in this model\(^{13}\). Besides the government’s inability to commit to repay \( \textit{ex ante} \), equilibrium multiplicity may also arise in this economy from the investors’ initial priors on the equilibrium bond price. For example, in one equilibrium the investors could start with an initial guess \( q^0 = 1/(1 + r) \) and, by monotonicity, obtain a high equilibrium bond price (low interest rate) and low default rate or in another equilibrium they could start with an initial prior \( q^0 = 0 \) and similarly obtain a low equilibrium bond price and high default rate, validating their initial priors\(^{14}\). For the quantitative exercise in this paper, I focus on the low price and high default equilibrium since this gives the upperbound default rate that we can attain from this model economy.

\[^{11}\text{See also Bassetto (2005) for other examples that follow this type of timing convention.}\]
\[^{12}\text{It is easy to show that their proof of the existence of competitive recursive equilibrium can also applied to the current equilibrium.}\]
\[^{13}\text{For example, consider an economy where the government makes its default decision first and sell its new debt later.}\]
\[^{14}\text{To my best knowledge, no one has yet formally proved this type of multiplicity in this environment. In the quantitative exercise, however, we can verify that the two extreme priors all converge monotonically to different equilibria.}\]
2.3 Self-fulfilling Debt Crisis

In this section, I study a set of stationary equilibria that are consistent with certain expectations of investors in the bond market. To model the strategic lending decision of investors, I explicitly model the bond market as a version of a competitive auction in which each investor submits a bond purchase bid consisting of a quantity and a price. When $B$ lies within a contingent interval of asset, $[B^f(y), B^s(y)]$, which I refer to as the self-fulfilling crisis zone, two possible equilibria arise: In one equilibrium, all investors purchase new bonds at a positive price and the government repays $B$. In another equilibrium, no investor purchase new bonds at a positive price and the government defaults on $B$. Among these two equilibria, the investors’ expectation selects the actual equilibrium of this economy. When $\xi = 0$, investors expect that the government will repay its old debt at the end of the period. Given this expectation, investors purchase new bond at a positive price and the government repays its debt in equilibrium, validating the investors’ expectation. When $\xi = 1$, each investor expects that the government will default on $B$ and therefore does not purchase $B'$ at any positive price. Then, the government indeed defaults in equilibrium, making the investors’ expectation self-fulfilling once again. The latter outcome is what I refer to as a self-fulfilling crisis.

Finally, I describe the equilibrium bond price and the agents’ equilibrium strategies over the asset space $B = [B, \bar{B}]$, which is divided by the crisis zone into the following three distinct intervals: $[B, B^f(y))$, $[B^f(y), B^s(y)]$, and $(B^s(y), \bar{B}]$. In an environment with no information friction and the perfectly observable sunspot variable acting as the only coordination device available, the realizations of $(y, \xi)$ and the level of asset $B$ uniquely determine the agents’ equilibrium policies in each one of these intervals\(^{15}\).

**Setup**  The government bond auction has two stages. First, the government issues new bonds of negative face value $B'$ in the auction, which the government commits to sell at any market price offered by investors. When issuing $B'$, the government perfectly predicts the equilibrium bond price and chooses $B'$ such that it solves the government’s maximization problem in (3) given $q(s, B')$.

Next, taking $B'$ as given, each investor submits his purchase bid $(b', q)$, which specifies the amount of new bonds the investor wishes to buy and the price he is willing to pay for

\(^{15}\)In an influential paper, Morris and Shin (1998) attribute the multiplicity in this type of models as an artifact of the common knowledge assumption and show that, when the information is perturbed by idiosyncratic noise, equilibrium multiplicity actually vanishes. See also Hellwig, Mukherji, Tsyvinski (2005) and Angelestos and Werning (2005) for other information-based models that respond to this point.
subject to the budget constraint (7)\textsuperscript{16}. When submitting their bids, investors perfectly predict the government’s default decision at the end of the current period as well as the likelihood of default associated with $B'$ in the next period. Given this information, investors make their bids perfectly competitively such that, conditional on the government’s repayment of $B$, they are indifferent between buying and not buying new bonds at the equilibrium price $q$.

Note that because each investor understands that his bid cannot by itself affect the equilibrium price, they have no incentive to consider any other possibly more complex bidding schemes. Furthermore, the ability to submit both the quantity and the price of new bonds ensures that the investors can always meet their budget constraint at any equilibrium market price.

In establishing the results in this section, two assumptions play crucial roles. First, an individual investor’s endowment in this economy has zero mass and hence a single investor by himself cannot affect the equilibrium bond price without violating his budget constraint (7). This assumption ensures that investors indeed face a coordination problem when bidding for new bonds, making self-fulfilling crisis possible as an equilibrium outcome of this economy. The second assumption is the investors’ belief that investors punish any deviator who lends to the currently defaulting government by seizing the proceeds from new bonds in the next period. Taking this out-of-equilibrium threat as credible, investors do not purchase $B'$ at any positive price when they expect the government to default on $B$ later in the current period\textsuperscript{17}.

**Self-fulfilling Crisis Zone** Next, I characterize the two cut-off values, $B_f(y)$ and $B^s(y)$. Let $v^r(s : q)$ and $v^d(s : q)$ be the government’s expected value of repayment and default in state $s = (y, B, \xi)$ at price $q$.

First, I define $B_f(y)$ to be an endowment-contingent value of asset that satisfies
\[
v^r(s : q) = v^d(s : q), \tag{11}
\]
where $q > 0$. Then, it follows that for any $B$ strictly less than $B_f(y)$ we have
\[
v^r(s : q) < v^d(s : q), \tag{12}
\]
from the fact that $v^r(s)$ is monotonically increasing in $B$ and that $v^d(s)$ is independent

\textsuperscript{16}Throughout this paper, I restrict attention to symmetric equilibria.

\textsuperscript{17}This assumption, of course, raises practical questions regarding how to actually implement this type of punishment, which I do not address in this paper.
of $B^{18}$. By definition, the cut-off value $B^f(y)$ denotes the upper limit of asset levels for which the government defaults independent of the investors’ lending decision: whenever the current realization of endowment places $B$ below $B^f(y)$, the government defaults in equilibrium.

Default in (12) differs from one triggered by investors’ self-fulfilling expectation in that fundamentals is the direct cause of default. When hit by a low endowment shock and burdened with high enough level of debt ($B < B^f(y)$), the government optimally chooses to default for higher consumption in the current period. Hence, default option effectively works as partial insurance scheme in this incomplete market setting. This type of insurance comes at the expense of higher default premium that correctly reflects the expected default probability associated with new bonds. In this sense, the default option resembles a call option of zero strike price for the government debt. Notice that the investors’ expectation plays no role when $B$ is less than $B^f(y)$ since the government’s decision to default in this case is independent of the investors’ purchase of new bond. The recent models of Arellano (2005), Aguiar and Gopinath (2006), and Yue (2005) all feature this type of default.

Next, define $B^s(y)$ to be the level of asset that satisfies

$$v^r(s : 0) = v^d(s : 0), \quad (13)$$

where

$$v^r(s : 0) = u(y + B^s(y)) + \beta \int_{\mathcal{Y} \times \Xi} v^u(y', 0, \xi') d\pi_{y'/y} d\pi_\xi.$$

The value $v^r(s : 0)$ denotes the government’s lifetime expected value of repaying its debt without any new borrowing from the investors. Then, it also follows that for any $B$ strictly greater than $B^s(y)$,

$$v^r(s : 0) > v^d(s : 0). \quad (14)$$

Hence, the cut-off value $B^s(y)$ defines the lower limit of assets for which the government is willing to repay its debt even when investors does not purchase $B'$ at a positive price. Hence, whenever $B$ is strictly greater than $B^s(y)$, the investors’ equilibrium strategy is to buy $B'$ at a positive price $q$.

**Proposition 1.** Let $B^f : \mathcal{Y} \rightarrow \mathcal{B}$ and $B^s : \mathcal{Y} \rightarrow \mathcal{B}$ be the cut-off values of asset defined in (11) and (13). Then, the following are true:

---

18This property of $v^d$ is of course a direct consequence of the exogenous default penalty in this paper and more than sufficient for (12) to hold.
(a) For every $y \in \mathcal{Y}$, there exist $B^f(y)$ and $B^s(y)$ in $\mathcal{B}$ such that $B^f(y) < B^s(y)$.

(b) If $y_2 > y_1$, $B^f(y_2) > B^f(y_1)$ and $B^s(y_2) > B^s(y_1)$


Proposition 1(a) and (b) follow immediately from the fact that the function $v^r$ is continuous and increasing with respect to $B$ and $y$. Intuitively, (a) implies that a government that is willing to repay its debt without a debt roll over is also willing to repay with a debt roll over. Proposition 1(b) states that self-fulfilling crisis zone is decreasing in output. While higher output raises the incentive to default ($v^d$), the increase in the government’s repayment incentive ($v^r$) dominates this effect and allows the government to sustain a higher level of debt (low $B$).

Coordination Problem  When the government’s level of debt, $B$, lies between the two cut-off values $B^f(y)$ and $B^s(y)$, the government’s decision to default on $B$ depends whether or not investors purchase $B'$ at a positive price $q$. Conditional on the state $s = (y, B, \xi)$ and the government’s newly issued debt $B'$, investors in turn face the following coordination problem: First, suppose an investor buys $B'$ at price $q > 0$. Then, he receives $-B > 0$ later in the period if the government repays $B'$, but loses $qB'$ if the government defaults on $B$. Suppose on the other hand that the investor does not buy $B'$ at any positive price and instead submit a competitive bid with $q = 0$. In this case, he earns $-B$ if the government repays $B'$, but loses $B$ if the government defaults on $B$. The following table summarizes the investor’s payoffs.

<table>
<thead>
<tr>
<th></th>
<th>Repay</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy : $q &gt; 0$</td>
<td>$-B$</td>
<td>$qB'$</td>
</tr>
<tr>
<td>Not Buy : $q = 0$</td>
<td>$-B$</td>
<td>0</td>
</tr>
</tbody>
</table>

Then, depending on the investors’ expectation regarding the government’s repayment policy at the end of the period, one of the following Nash equilibria ensues:

$$(Repay, \text{ Buy}) : \quad v^r(y, B, 0 : q) \geq v^d(y, B, 0 : q), \quad (15)$$

$$(Default, \text{ Not Buy}) : \quad v^r(y, B, 1 : 0) < v^d(y, B, 1 : 0), \quad (16)$$

where $q > 0$ in (15). In (15), the investors observe $\xi = 0$ and expect the government to repay its debt. In equilibrium, the investors buy new bonds at a positive price and the government indeed repays its old debt $B$, validating the investors’ expectation. In (16), on the other hand, the investors observe $\xi = 1$ and expect the government to default. Hence in
equilibrium, the investors do not buy new bond and the government defaults, making the investors’ expectation self-fulfilling once again.

Notice that the equilibrium outcome (16) is Pareto-dominated by (15) and hence constitutes a coordination failure: investors could all be better off by making loans to the government than not when \( B \) is between \( B^f(y) \) and \( B^s(y) \). In this respect, this type of default essentially differs from fundamentals-driven default in (12), which arises as the unique optimal strategy for the government no matter what the investors’ lending decision.

A typical default episodes in this model economy is as follows: Preceding default, the government receives a series of high endowment shock and accumulates a large level of debt. Then, the current output shock realization places the government either in the crisis zone \([B^f(y), B^s(y)]\) or in the default zone \([\underline{B}, B^f(y)]\). The government will then optimally default only when investors expect the government to default inside the crisis zone or when the value of output places the government within the default zone.

**Equilibrium Policies and Price** Next, I describe the equilibrium policy of the government and investors and the equilibrium bond price over the asset space \( \mathcal{B} = [\underline{B}, \bar{B}] \).

For every state \( s = (y, B, \xi) \) and given the government’s optimal debt policy \( B'(s) \) from (3), the government’s equilibrium default decision \( d(s) \) is as follows:

\[
d(s) = \begin{cases} 
1 & : B \in [\underline{B}, B^f(y)], \\
\xi & : B \in [B^f(y), B^s(y)], \\
0 & : B \in (B^s(y), \bar{B}]. 
\end{cases}
\]

Consistent with the government’s equilibrium debt policy \( B'(s) \) and default decision \( d(s) \), the individual investors’ equilibrium lending policy is given by

\[
b'(s) = \begin{cases} 
0 & : B \in [\underline{B}, B^f(y)], \\
(1 - \xi)B'(s) & : B \in [B^f(y), B^s(y)], \\
B'(s) & : B \in (B^s(y), \bar{B}]. 
\end{cases}
\]

In equilibrium, investors are not willing to purchase new debt at a positive price when \( \xi = 1 \) within the crisis zone and thus the value of \( b'(s) \) is actually irrelevant.

To describe the equilibrium bond price \( q(s, B') \), it is sufficient to characterize the expected default probability \( \phi(s, B') \) associated with new debt \( B' \). From the equilibrium policies...
of the government and investors, we have

$$
\phi(s, B') = \begin{cases} 
\int_y d(y', B', 0) d\pi_{y'|y} : B' \in [B, B^f(y')), \\
\eta \int_y d(y', B', 1) d\pi_{y'|y} : B' \in [B^f(y'), B^s(y')], \\
0 : B' \in (B^s(y'), \bar{B}]
\end{cases}
$$

where $$\eta$$ is the exogenous probability that $$\xi = 1$$. The intuition for $$\phi(s, B')$$ is as follows: Suppose the government has current endowment realization $$y$$ and chooses the level of new debt $$B'$$. Then, the government faces probability $$\int_y d(y', B', 1) d\pi_{y'|y}$$ of being placed within the crisis zone in the next period and probability $$\eta$$ of not being able to borrow from investors, in which case the government will choose to default. Similarly, the government has probability $$\int_y d(y', B', 0) d\pi_{y'|y}$$ of having $$B'$$ placed below the cut-off value $$B^f(y)$$, in which case the governemnt will surely choose to default irrespective of the realization of $$\xi$$.

The theoretical innovation of this paper relative to the previous studies is summarized by the liquidity premium term $$\eta \int_y d(y', B', 1) d\pi_{y'|y}$$. Notice that the current model includes Aguiar and Gopinath’s (2006) model as a special case in which $$\eta = 0$$.

To gain further insight for the model, I define a default set as follows:

$$D(B, \xi) = \left\{ y \in \mathcal{Y} | \ v^r(s : q) < v^d(s : q) \right\},$$

where

$$D(B, 0) = \left\{ y \in \mathcal{Y} | \ v^r(y, B, 0 : q) < v^d(y, B, 0 : q) \right\},$$

$$D(B, 1) = \left\{ y \in \mathcal{Y} | \ v^r(y, B, 1 : 0) < v^d(y, B, 1 : 0) \right\}.$$

A default set $$D(B, \xi)$$ defines the set of endowments for which the government chooses to default for a given level of asset $$B$$ and a sunspot realization $$\xi$$. For example, the government always defaults whenever $$D(B, \xi) = \mathcal{Y}$$ and always repays whenever $$D(B, \xi) = \emptyset$$. The following proposition establishes the properties of the default set.

**Proposition 2.** (a) For every $$B \in \mathcal{B}$$, $$D(B, 0) \subseteq D(B, 1)$$.

(b) For $$B^2 \geq B^1$$, $$D(B^2, \xi) \subseteq D(B^1, \xi)$$, $$\xi = 0, 1$$.

(c) If $$y_2 > y_1$$ and $$y_2 \in D(B, \xi)$$, then $$y_1 \in D(B, \xi)$$, $$\xi = 0, 1$$.

**Proof.** See Appendix

Intuitively, Proposition 2(a) implies that if the government decides to default when it is able to sell new debt at a positive price $$q$$, the government will also default when it cannot...
sell new debt at a positive price as well. Note that we have \( D(B, 0) = D(B, 1) \) if, and only if, \( D(B, 0) = D(B, 1) = \emptyset \) or \( \emptyset \): either when \( B \) is so low that the government always defaults for every \( y \) regardless of new borrowing or so high that it always repays for all \( y \)'s. Proposition 2(b) implies that the default sets, for both \( \xi = 0 \) and 1, shrinks as the level of asset increases\(^{20}\). As result, the expected default probability \( \phi(s, B') \) decreases with \( B' \). Finally, Proposition 2(c) implies that the government’s default incentive decreases with output. This is because while a higher output raises the government’s incentive to default, it increases the repayment incentive relatively more.

3 Quantitative Analysis

For quantitative analysis, I calibrate the model parameters to match the business cycle statistics of Argentina. According to Reinhard, Rogoff, and Savastana (2003), Argentina defaulted on its debt five times in the last 180 years, including the last crisis episode in 2001. Furthermore, its business cycle exhibits the typical features of emerging economies such as strongly countercyclical and highly volatile real interest rates and net exports (Neumeyer and Perri (2005)). Finally, since most of previous similar studies in the literature also conduct quantitative analyses on Argentina, the result in this paper are directly comparable to theirs. The sum of all these factors makes the country an ideal candidate for this research.

3.1 Calibration

I define one period as a quarter. The riskless world interest rate \( r \) is set to 1% from the average yield of 3-month U.S. T-bill. The domestic representative household’s within-period utility function has the standard power utility form

\[
    u(c) = \frac{c^{1-\sigma}}{1-\sigma},
\]

where \( \sigma \) is the risk aversion coefficient. In the benchmark analysis, I set \( \sigma = 2 \). The time discount factor \( \beta \) is set to 0.8. While this is a very low value relative to the ones used in most open economy business cycle literature, it is not unusual in this class of endogenous default risk models.

In modelling endowment process, this paper follows Aguiar and Gopinath’s (2006) work, which finds that a non-stationary endowment process with a stochastic trend is able to

\[^{20}\text{See also Chatterjee, Corbae, Nakajima, and Rios-Rull (2005) and Arellano (2005) for similar results.}\]
better replicate the strongly countercyclical net exports and the high default rate than one with a constant trend. Specifically, I model output $y_t$ as follows:

$$y_t = \exp(g_t) y_{t-1},$$

where $g_t$ is the output growth rate that follows an AR(1) process

$$g_t = (1 - \rho_g) \bar{g} + \rho_g g_{t-1} + \varepsilon_t^g,$$

with $|\rho_g| < 1$, $\varepsilon_t^g \sim N(0, \sigma^2_g)$ and $\bar{g}$ denotes the long-run mean output growth rate. I estimate this process using an HP-filtered series of the seasonally-adjusted quarterly real GDP from 1980Q1 to 2005Q4 from the Ministry of Finance (MECON) and set $\bar{g} = 1.0051$, $\rho_g = 0.12$, and $\sigma_g = 0.305$. For the numerical analysis, I make the model stationary by detrending each variable using the previous period’s output $y_{t-1}$\textsuperscript{21}.

As for the value of return probability $\psi$ and direct output cost $\lambda$, I follow Aguiar and Gopinath’s (2006) choice of 0.1 and 0.02, respectively. For return probability, Aguiar and Gopinath’s (2006) rely on Gelos, Sahay, and Sandleris (2002) that document defaulting countries throughout the 80’s and the 90’s could not regain access to international capital markets in less than 3 years on average\textsuperscript{22}. As for the direct output cost of default, they adopt the Struzenegger’s (2002) estimate using a panel of 100 countries in the 80’s.

To investigate the quantitative significance of self-fulfilling default on the Argentina’s business cycle, I simulate the model economy for different values of the sunspot probability, $\eta$. While the model itself does not impose any restriction on the value of this parameter, the historical default rate of 0.7% for Argentina (Reinhart, Rogoff, and Savastano (2003)) sets an empirical upperbound on the value of $\eta$. In this paper, I conduct analyses for four different values of $\eta$: $[0, 0.001, 0.003, 0.005]$. In particular, our focus will be on the case of $\eta = 0.005$, which turns out to be the value that closely matches the actual default rate of 0.7%. Table 2 summarizes the calibration result.

### 3.2 Simulation Results

I solve the calibrated model numerically using the computational procedure that I describe in Appendix. To obtain model statistics for Argentina, I calculate the average of 100

\textsuperscript{21}See Aguiar and Gopinath (2006) for detail.

\textsuperscript{22}It should be noted that there exists a large variance in $\psi$ across different time periods considered. In the 90’s, for example, they find that defaulting countries usually could exit financial autarky in less than a year, while in the 80’s it was about 6 years on average. I emphasize, however, that the quantitative significance of sunspot shock in this model is robust to the particular choice of this parameter value.
Table 1: Model Parameters

<table>
<thead>
<tr>
<th>Section A.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Aversion</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>Risk-free Rate</td>
<td>$r$</td>
</tr>
<tr>
<td>Return Probability</td>
<td>$\psi$</td>
</tr>
<tr>
<td>Output Loss</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Time Discount Factor</td>
<td>$\beta$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section B.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Output Growth</td>
<td>$\mu_g$</td>
</tr>
<tr>
<td>Output Std. Dev.</td>
<td>$\sigma_g$</td>
</tr>
<tr>
<td>Output AR(1) Coef.</td>
<td>$\rho_g$</td>
</tr>
</tbody>
</table>

| Sunspot Probability | $\eta$ | [0, 0.001, 0.003, 0.005] |

simulations with 50 observations in each. Each simulation runs the model economy for 10000 periods and only the last 100 periods are sampled for calculation.

Table 2 presents the actual and model business cycle statistics for Argentina from 1980 to 2005. The actual output and consumption data come from the Ministry of Finance (MECON) and are seasonally-adjusted quarterly real series. The log of both series have been Hodrick-Prescott filtered with parameter 1600. The net export variable is defined as the share of trade balance to GDP. The real interest rate series come from Neumeyer and Perri (2005) and are calculated by adding the 3-month U.S. T-bill rate to the Argentina EMBI (Emerging Market Bond Index) series after adjusting for inflation. The model output and consumption series from simulations are treated in the same manner as the actual series and the model interest rate is calculated from the discount bond price as $R = 1/q$.

The model economy successfully replicates several features of Argentina’s business cycle. First, compared to the baseline model statistics with no self-fulfilling debt crises ($\eta = 0$), the existence of self-fulfilling debt crises significantly improves the volatilities of net exports and real interest rates relative to the data. For example, the sunspot model with $\eta = 0.005$ can account for about 76% of the actual variation in net exports. Furthermore, the relative volatility of consumption with respect to output ($\sigma(c)/\sigma(y)$) increases from 1.08 to 1.12, bringing the model closer to the actual ratio of 1.14. The model, however, is less successful with interest rate series and explains only about 16% of the volatility in data. Such a low volatility of interest rate is not uncommon among this type of endogenous default risk models.
Second, consistent with data, model net exports and interest rates are countercyclical and positively correlated with each other. This is because, with output process featuring a stochastic trend, a high output shock today implies even higher expected outputs in the subsequent periods and this persistency in output growth rate induces the government to borrow more in good times rather than in bad times, leading to countercyclical net exports.

To understand the movement of interest rate, note that higher borrowing in good times can have two opposing effects on the interest rate. On one hand, higher borrowing implies higher default premium, which we can illustrate in Figure 2 as a leftward movement along the bond price curve. So, this effect works to make interest rates to be even more negatively correlated with output. On the other hand, a high output shock implies lower levels of default premiums will be applied for all values of asset with positive default probability. In Figure 2, this effect corresponds to a leftward shift of bond price curve. The factor that determines which one of these two effects dominates the other is the slope of the bond price curve²³. If the slope is "too" steep, the increase in the default premium associated with higher borrowing becomes dominant and the equilibrium interest rate will rise and be procyclical. Aguiar and Gopinath (2006) show quantitatively that an output process with

²³For a formal argument, see Aguiar and Gopinath (2006) and Arellano (2005).
stochastic trend ensures that the slope moderately steep enough such that interest rates are countercyclical.

The self-fulfilling expectation model, however, also exhibits a counterfactual feature that as we increase the chance of coordination failure, $\eta$, the degree of negative correlation with output decreases further from data for both net exports and interest rates. To understand why net exports become less countercyclical, note that the introduction of liquidity premium in this model makes the government borrowing significantly more costly. As result, the government to consume and borrow relatively less in high output state, leading to less countercyclical net exports. This fact is the most evident from the change of average debt service to GDP ratio. Compared to the no self-fulfilling expectation case ($\eta = 0$), the self-fulfilling expectation model with $\eta = 0.005$ reduces the debt service payment of the economy by nearly 2\% on quarterly basis relative to GDP. At the quarterly interest rate of 2\%, this implies a decrease in the average total debt to GDP ratio of almost 100\%. As for the interest rates, note that reduction in the government borrowing in the absence of liquidity premium would actually make the interest rate even more countercyclical since lower borrowing in high output state implies lower interest rate. However, because of the addition of liquidity premium, lower borrowing in high output state is actually associated to slightly higher interest rate, implying less countercyclical interest rates.

The model default rate with no coordination failure is 0.22\%, which is very close to what Aguiar and Gopinath (2006) and other similar studies generate\(^{24}\). Furthermore, the default rate increases one-to-one with $\eta$ and self-fulfilling default rate. This implies that, although the government borrows less in good times by the introduction of liquidity premium, the government’s default incentive is not significantly affected. Consequently, the rate of fundamentals-driven default remains around at a constant rate of around 0.2\% and the increase in the probability of coordination failure is translated exactly one-to-one into the model default rate, an evidence that the government’s asset position lies mostly within the crisis zone. Note also that the bond spread matches the default rate one-to-one because international investors are risk neutral\(^{25}\).

Figure 1 shows the self-fulfilling debt crisis zone with $\eta = 0.005$ and illustrates how each interval is determined for each pair of $(y, B)$. For each level of output growth rate,

\(^{24}\)Arellano’s (2006) model is able to match the Argentina’s historical default rate of 3\% using a more flexible form of output cost. In this paper, I use a constant output cost parameter $\lambda$ as in Aguiar and Gopinath (2006) to ensure that simulation results in both papers are directly comparable with each other.

\(^{25}\)See Yue (2006) for a model that breaks this one-to-one relationship between default rate and bond spread by modelling endogenous debt recovery.
Figure 1: The Self-fulfilling Debt Crisis Zone
the segment in the figure represents the corresponding self-fulfilling crisis zone: for any level of asset below the left end point of the interval, the government defaults regardless of endowment realization \((v^r(s:q) < v^d(s))\). Similarly, for the asset levels higher than the right end point of the interval, the government always repays its debt regardless of roll over \((v^r(s:0) > v^d(s))\). Notice that, in order for self-fulfilling crisis to occur, quite a large level of debt payment to output ratio is required: excluding the unlikely extreme values of output realization, the government must have between 12% to 18% of debt payment to output share to be exposed to self-fulfilling default risk. This is a large number compared to Argentina’s historical average debt payment to output ratio\(^{26}\) of 6% from 1980 to 2005. As I will show later, the factor contributing to such a large debt payment ratio in this model is the high direct output cost of default. With \(\lambda = 1\%\), for example, this ratio falls to 7.4%.

Figure 2 shows the discount bond price and the default probability for the highest and lowest endowment. The bond price is increasing in the level of asset and eventually reaches zero below a certain cutoff value of asset. That cutoff value therefore sets the endogenous borrowing constraint for the government borrowing. Notice that for the values of asset within the crisis zone the bond price drops slightly to reflect the liquidity premium inside the crisis zone. The default probability decreases monotonically with the level of asset and

\(^{26}\)Debt payment ratio is calculated from World Bank data and includes short-term debt payment and long-term debt interest payment.
is perfectly symmetric to the bond price since the bond price is modelled as a simple linear function of this probability.

3.3 When Do Countries Default?

A surprising aspect of sovereign defaults among emerging countries is the fact that defaults occur not only in economic downturns, but also even when the economic fundamentals are relatively strong. In a recent empirical study on sovereign defaults among 175 debtor countries for the period 1820-2004, Tomz and Wright (2006) document that the correlation between output and defaults is historically negative, but find the degree of relationship to be remarkably weak (Table 3). Conditional on default, output is on average $-1.6\%$ below Hodrick-Prescott trend and defaults occur in about 62% of the periods below the trend. The benchmark model with no self-fulfilling expectation, however, predicts a far more strong negative relationship between output and defaults: output is on average $-7.9\%$ below trend conditional on default and defaults occur in about 88% of the periods in which output is below the trend. The reason for this excessively strong negative correlation between defaults and outputs in these previous models is because, in these models, defaults can occur only in response to a bad output shock.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Data</th>
<th>No Self-fulfilling Def.</th>
<th>With Self-fulfilling Def.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>% Mean Deviation from Trend</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In first period of default</td>
<td>-1.6</td>
<td>-7.9</td>
<td>-2.5</td>
</tr>
<tr>
<td>In periods of default</td>
<td>-1.4</td>
<td>-4.5</td>
<td>-0.6</td>
</tr>
<tr>
<td>In periods of non-default</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>% of Defaults Below Trend</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In first period of default</td>
<td>61.5</td>
<td>87.9</td>
<td>61.4</td>
</tr>
<tr>
<td>In periods of default</td>
<td>56.2</td>
<td>72.9</td>
<td>52.9</td>
</tr>
<tr>
<td>In periods of non-default</td>
<td>47.3</td>
<td>49.3</td>
<td>49.4</td>
</tr>
</tbody>
</table>

Source: Tomz and Wright (2006)

Table 3: The Relationship Between Output and Default

The introduction of the coordination failure in this paper improves the model’s prediction by allowing defaults to arise not only in bad times, but also in good times. With sunspot probability $\eta$ of 0.015, which is the value that allows us to match the historical default rate

$27$ Following Tomz and Wright (2006), I use the annualized values of parameters for computing these numbers: $\sigma = 2, \psi = 0.34, r = 0.04, \lambda = 0.02, \beta = 0.41, \mu_g = 1.02, \rho_g = 0.0008, \sigma_g = 0.07.$
in Tomz and Wright (2006)'s dataset, the model performs surprisingly well in reproducing their historical output-default relationship. The country typically defaults in recessions of $-2.5\%$ below trend and stays in default with output growth of $-0.6\%$. Moreover, defaults occur in about 61\% of the periods with output below trend and output stays below trend for about 53\% of the default periods.

![Distribution of Self-fulfilling Default](image1)

![Distribution of Output-driven Default](image2)

**Figure 3: Self-fulfilling Default vs. Output-driven Default**

Figure 3 displays the distribution of self-fulfilling and output-driven default over the output deviations from Hodrick-Prescott trend. While both distributions are symmetric, their first moments are substantially different: the mean of self-fulfilling default distribution is on the HP-trend, while that of output-driven default distribution is well-below the trend. The reason self-fulfilling default distribution has mean zero is because the belief of investors is independent of output and the government’s asset position is almost always within the crisis zone along the simulated paths of model economy. As result, self-fulfilling default is equally likely with both low and high output realizations.

### 3.4 Sensitivity Analysis

Table 4 presents the result of sensitivity analysis for some of the key parameters in the model economy with $\eta = 0.005$: $\sigma, \beta, \psi, \lambda,$ and $r$.

First, the model statistics are overall robust to the changes in the household’s degree of risk aversion $\sigma$ and of impatience $\beta$. In particular, the volatility of interest rate turns out to be almost unaffected by changes in the degree of risk aversion, which I find to be a bit surprising. However, note that as we make the household more patient by increasing the
Table 4: Sensitivity Analysis

<table>
<thead>
<tr>
<th>%, quarterly</th>
<th>Default Rate</th>
<th>DebtService Output</th>
<th>mean(R)</th>
<th>σ(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark (η = 0.005)</td>
<td>0.67</td>
<td>15.3</td>
<td>0.69</td>
<td>0.58</td>
</tr>
<tr>
<td>Household</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ = 1.5</td>
<td>0.67</td>
<td>15.1</td>
<td>0.69</td>
<td>0.58</td>
</tr>
<tr>
<td>σ = 10</td>
<td>0.67</td>
<td>17.5</td>
<td>0.68</td>
<td>0.61</td>
</tr>
<tr>
<td>β = 0.75</td>
<td>0.67</td>
<td>14.8</td>
<td>0.69</td>
<td>0.59</td>
</tr>
<tr>
<td>β = 0.85</td>
<td>0.54</td>
<td>15.8</td>
<td>0.55</td>
<td>0.34</td>
</tr>
<tr>
<td>Default Cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ψ = 0.2</td>
<td>0.55</td>
<td>9.26</td>
<td>0.57</td>
<td>0.25</td>
</tr>
<tr>
<td>ψ = 0.5</td>
<td>0.56</td>
<td>4.14</td>
<td>0.58</td>
<td>0.18</td>
</tr>
<tr>
<td>λ = 0.01</td>
<td>0.54</td>
<td>7.4</td>
<td>0.56</td>
<td>0.36</td>
</tr>
<tr>
<td>λ = 0.03</td>
<td>0.66</td>
<td>23.8</td>
<td>0.68</td>
<td>0.51</td>
</tr>
<tr>
<td>Riskless Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r = 0.02</td>
<td>0.56</td>
<td>13.8</td>
<td>0.59</td>
<td>0.51</td>
</tr>
<tr>
<td>r = 0.03</td>
<td>0.66</td>
<td>12.4</td>
<td>0.69</td>
<td>0.57</td>
</tr>
</tbody>
</table>

value of β from 0.8 to 0.85, the default rate drops from 0.67% to 0.54%. After subtracting the rate of self-fulfilling default, we can verify that the fundamentals-driven default rate decreases substantially from 0.21% to 0.08%, which reflects the fact that the more patient government finds default option less attractive. investors, on their side, are willing to lend more to the patient government at lower costs.

Second, the model is very sensitive to the changes in the cost of default. Shortening the duration of financial exclusion from average 10 (ψ = 0.1) to 2 (ψ = 0.5) quarters reduces the government’s borrowing by more than 10%. This is because investors anticipate that default option has now become more attractive to the government and thus raise the cost of borrowing to compensate themselves for this higher default risk. As result, the government borrows less and defaults less. Similarly, decreasing the direct output cost of default implies higher default incentive and thus reduces the government’s debt limit. In both cases, the volatility of interest rates decreases.

Finally, higher riskless rate implies higher borrowing cost and hence reduces the government’s debt-to-output ratio. However, because the increase in the interest rate reflects an exogenous increase in riskless rate and not the government’s higher default incentive, the government’s access to international capital is not as severely restricted as in the case of
reducing the cost of default.

4 Concluding Remark

This paper studies the business cycle implications of sovereign default risk in an environment in which defaults can occur not only from bad output shocks, but also from coordination failures among international investors. The economic condition that gives rise to the coordination failure in this paper is not an artifact of an hypothetical parameter value, but an actual feature of an economy that we can directly calculate from the country’s GDP and external debt data.

In a quantitative analysis of Argentina, the model is able to reproduce several features of the economy: defaults are only weakly correlated with outputs and consumption is significantly more volatile than output, consistent with the data. The model also predicts that even a small increase in the chance of coordination failure can lead to substantial reduction in the country’s borrowing through its effect on the country spread.

This paper, however, cannot replicate the high volatility of country spread observed in emerging market data.
References


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Appendix A  Computation

I solve the model numerically using discrete space method similar to Aguiar and Gopinath (2006). I discretize the endowment space into 25 equally spaced grids using the quadrature method proposed by Hussey and Tauchen (1991). I discretize the asset space $\mathcal{B} = [B, \bar{B}]$ into 400 grids and set both limits such that they never bind in the simulations.

The computational algorithm is the following value function iteration:

1. Guess an initial bond price schedule $q_0 = 1/(1+r)$.
2. Taking $q_0$ as given, solve for the value functions $\{v_u^0, v_r^0, v_d^0, v_d^0\}$, the government’s debt policy $B_0'$.
3. From $\{v_r^0, v_d^0\}$, obtain the default set $D_0(B, \xi)$.
4. Using the Markov transition matrix for endowment $\Pi_y$, compute the default probability $\phi_0$ and the new bond price schedule $q_1 = \frac{1-\phi_0}{1+r}$.
5. Repeat step 1–4 $n$ times such that $|q_n - q_{n-1}| < \varepsilon$, where $\varepsilon$ is a very small positive number.

Appendix B  Proofs

Proof of Proposition 2. (a) Let $y^0$ be an element in $D(B, 0)$. By definition, we have $u(y^0 + B - q(y^0, B')B') + \beta \int_{\mathcal{Y} \times \Xi} v^u(y', B', \xi') d\pi_y y' d\pi_\xi < u(y^0) + \beta \int_{\mathcal{Y}} v^u(y') d\pi_y y'$ for every $B' \in \mathcal{B}$ and the price schedule $q(y, B', \eta)$. Since $-q(y, B', \eta)B' > 0$ and $v^u$ is strictly increasing and continuous (Chatterjee, Corbae, Nakajima, Rios-Rull (2005), p. 12, Thm 1), it follows that $u(y^0 + B) + \beta \int_{\mathcal{Y} \times \Xi} v^u(y', 0, \xi') d\pi_y y' d\pi_\xi < u(y^0 + B - q(y^0, B')B') + \beta \int_{\mathcal{Y} \times \Xi} v^u(y', B', \xi') d\pi_y y' d\pi_\xi < u(y^0) + \beta \int_{\mathcal{Y}} v^u(y') d\pi_y y'$, which implies $y^0 \in D(B, 1)$.

For (b) and (c), Arellano (2005) provides the proof for the case of $\xi = 0$. The proof for $\xi = 1$ is trivial. 

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